

Digital Input	Binary Output		
D ₇ D ₆ D ₅ D ₄ D ₃ D ₂ D ₁ D ₀	Q ₂	Q ₁	Q ₀
0 0 0 0 0 0 0 1	0	0	0
0 0 0 0 0 0 1 x	0	0	1
0 0 0 0 0 1 x x	0	1	0
0 0 0 0 1 x x x	0	1	1
0 0 0 1 x x x x	1	0	0
0 0 1 x x x x x	1	0	1
0 1 x x x x x x	1	1	0
1 x x x x x x x	1	1	1

$$Q_1 = \sum(3, 4, 6, 7) = \bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 \bar{D}_3 D_2 + \bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 D_3 + \bar{D}_7 D_6 + D_7$$

$$= \bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 (D_2 + D_3) + (\bar{D}_7 D_6 + D_7)$$

$$\bar{D}_3 D_2 + D_3 = \underbrace{(\bar{D}_3 + D_3)(D_2 + D_3)}_{(D_2 + D_3)(1)} = D_2 + D_3$$

$$= \bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 (D_2 + D_3) + D_6 + D_7$$

$$\bar{D}_7 D_6 + D_7 = \underbrace{(\bar{D}_7 + D_7)(D_6 + D_7)}_{(D_6 + D_7)(1)} = D_6 + D_7$$

$$\bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 (D_2 + D_3) + D_6 + D_7$$

$$= \bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 (D_2 + D_3) + D_6 + D_7$$

$$= (1) \cdot (\bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 (D_2 + D_3) + D_6 + D_7)$$

$$= \bar{D}_7 \bar{D}_6 \bar{D}_5 \bar{D}_4 (D_2 + D_3) + D_6 + D_7$$

$$Q_1 = ((\bar{D}_3 \bar{D}_4) \cdot (D_2 + D_3)) + D_6 + D_7$$

$$Q_2 = \sum(4, 5, 6, 7) = \bar{D}_1 \bar{D}_2 \bar{D}_3 D_4 + \bar{D}_1 \bar{D}_2 D_3 + \bar{D}_1 D_2 + D_1$$

$$= \bar{D}_1 \bar{D}_2 (\bar{D}_3 D_4 + D_3) + (\bar{D}_1 D_2 + D_1)$$

$$= \bar{D}_1 \bar{D}_2 (D_3 + D_4) + D_2 + D_1$$

$$\bar{D}_3 D_4 + D_3$$

$$= (D_3 + \bar{D}_3) \cdot (D_3 + D_4)$$

$$= (1) \cdot (D_3 + D_4)$$

$$= D_3 + D_4$$

$$Q_2 = ((\bar{D}_1 \bar{D}_2) \cdot (D_3 + D_4)) + D_2 + D_1$$

$$\bar{D}_1 D_2 + D_1$$

$$= (D_1 + \bar{D}_1) \cdot (D_1 + D_2)$$

$$= (1) \cdot (D_1 + D_2)$$

$$= D_1 + D_2$$