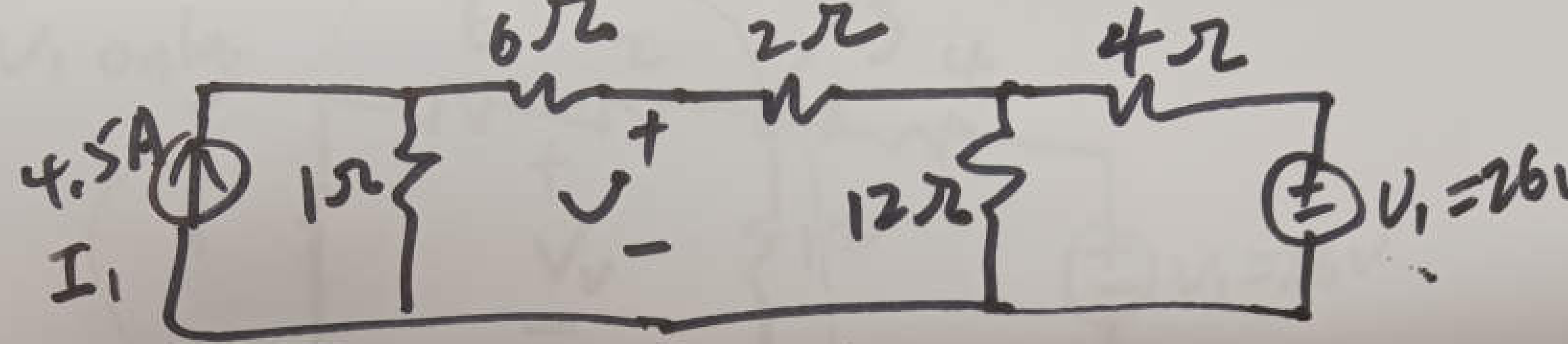
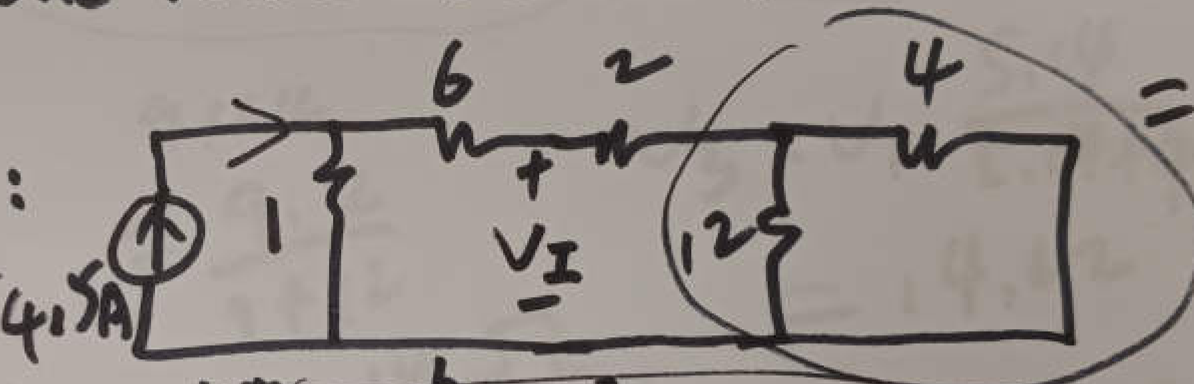
$$
\begin{cases}
-100V + (i_1 - i_2)\cdot 5 + (i_1 - i_3)\cdot 10 + i_1 \cdot 2 = 0 \\
i_2 \cdot 2 + V_{cs} + (i_2 - i_1)\cdot 5 = 0 \\
(i_3 - i_1)\cdot 10 - V_{cs} + i_3 \cdot 20 = 0 \\
i_3 - i_2 = 1.2 i_b = 1.2 i_1
\end{cases}
$$

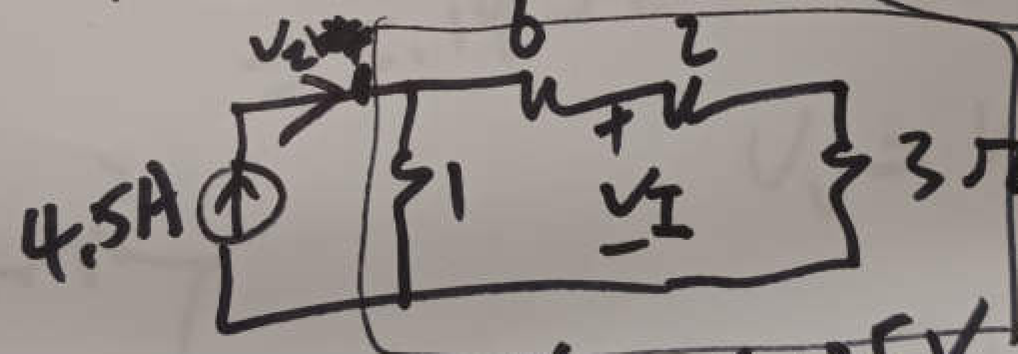


apply one source at a time:

I_1 only:



$$= \frac{4 \cdot 12}{4+12} = \frac{48}{16} = 3\,\Omega$$

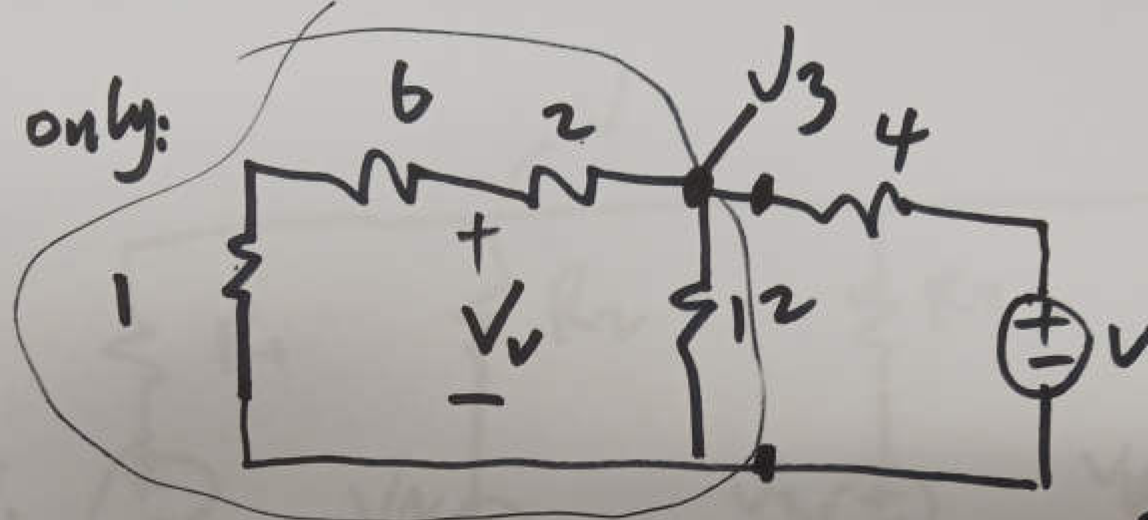


$$= 1 // 11 = \frac{1 \cdot 11}{1+11} = \frac{11}{12}$$

$$V_2 = 4.5 \cdot \frac{11}{12} = 4.125\,V$$

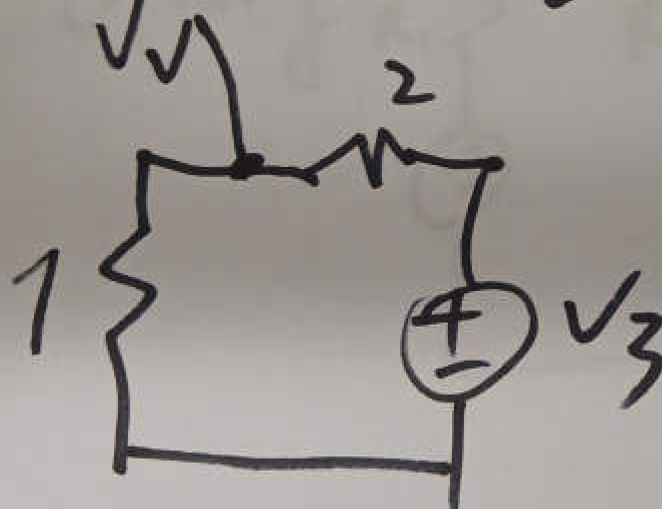
$$V_I = 4.125 \cdot \frac{5}{6+5} = 1.875\,V$$

V_1 only:



$$9 \| 12 = \frac{9 \cdot 12}{9+12} = 5.14\,\Omega$$

$$V_3 = V_1 \cdot \frac{5.14}{5.14+4} = 14.62$$

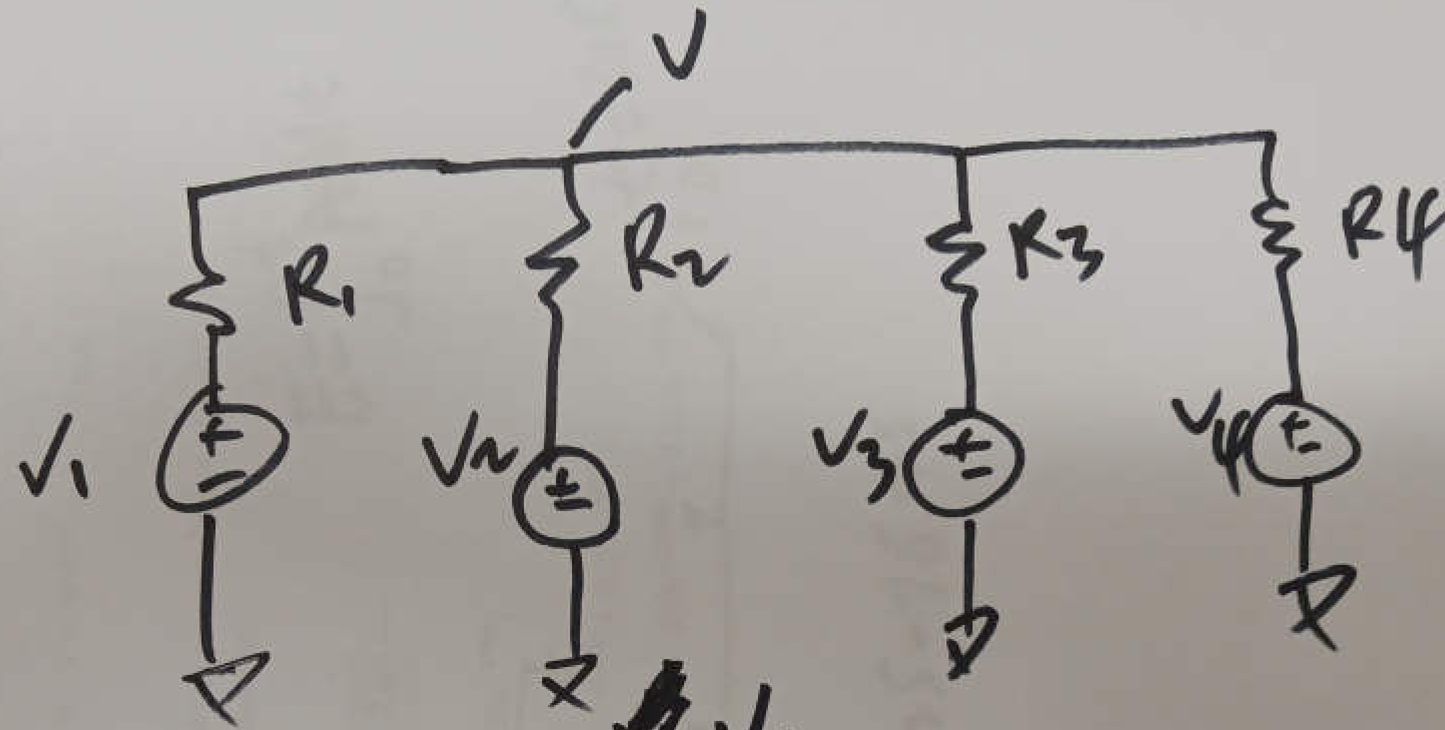


$$V_v = V_3 \cdot \frac{7}{9} = 11.37\,V$$

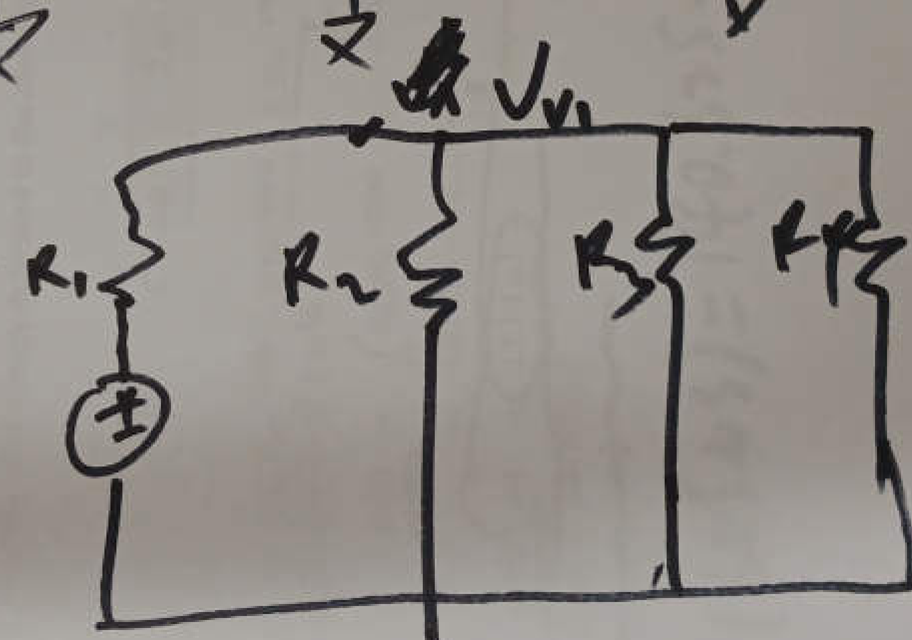
so overall v is

$V_I + V_v = 1.875V + 11.37V =$

4



V_1 only



Colorado School of Mines

Midterm Exam I

Wednesday 10/02/2019 - 2:00 PM – 50 minutes – closed book, closed notes

Q1.

The voltage and current at the terminals of the circuit element shown are zero for $t < 0$. For $t \geq 0$ they are

$$v = 50e^{-1600t} - 50e^{-400t}\ \text{V},\quad i = 5e^{-1600t} - 5e^{-400t}\ \text{mA}.$$

a) Find the power at $t = 625\ \mu s$.

b) How much energy is delivered to the circuit element between 0 and 625 μs?

c) Find the total energy delivered to the element.

$E = \int_0^t p(t)dt$

[a] $p = vi = 0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t}$;

$p(625\ \mu\text{s}) = 42.2$ mW.

[b] $$w(t) = \int_0^t (0.25e^{-3200t} - 0.5e^{-2000t} + 0.25e^{-800t})$$

$$= 140.625 - 78.125e^{-3200t} + 250e^{-2000t} - 312.5e^{-800t}\ \mu\text{J};$$

$\Big|_0^t = 625\ \mu s$

$w(625\ \mu\text{s}) = 12.14\ \mu\text{J}.$

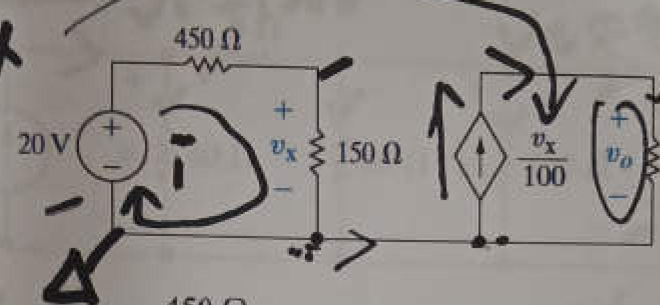
[c] $w_{\text{total}} = 140.625\ \mu\text{J}.$

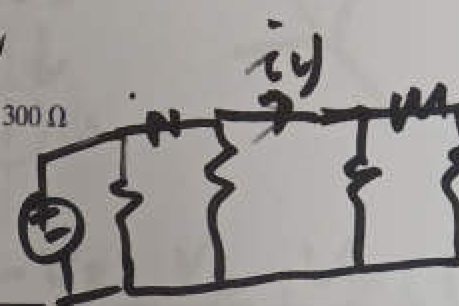
$W(625\mu s) = 140.625 - 78.125$

Q2.

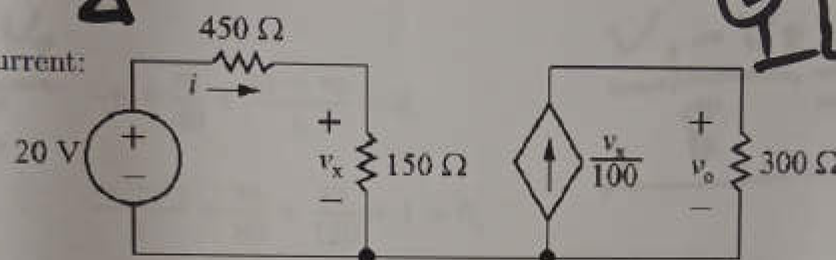
For the circuit shown below, find ~~v_o and the~~ total power absorbed in the circuit.

$$20 \cdot \frac{150}{450+150} = V_x$$





Label unknown current:



$$-20 + i \cdot 450 + i \cdot 150 = 0$$

$$-20 + 450i + 150i = 0 \qquad \text{(KVL and Ohm's law);}$$

so $\quad 600i = 20 \quad \rightarrow \quad i = 33.33\,\text{mA}.$

$$v_x = 150i = 150(0.0333) = 5\,\text{V} \qquad \text{(Ohm's law);}$$

$$v_o = 300\left(\frac{v_x}{100}\right) = 300(5/100) = 15\,\text{V} \qquad \text{(Ohm's law).}$$

Calculate the power for all components:

$$p_{20\text{V}} = -20i = -20(0.0333) = -0.667\,\text{W};$$

$$p_{\text{d.s.}} = -v_o\left(\frac{v_x}{100}\right) = -(15)(5/100) = -0.75\,\text{W};$$

$$p_{450} = 450i^2 = 450(0.033)^2 = 0.5\,\text{W};$$

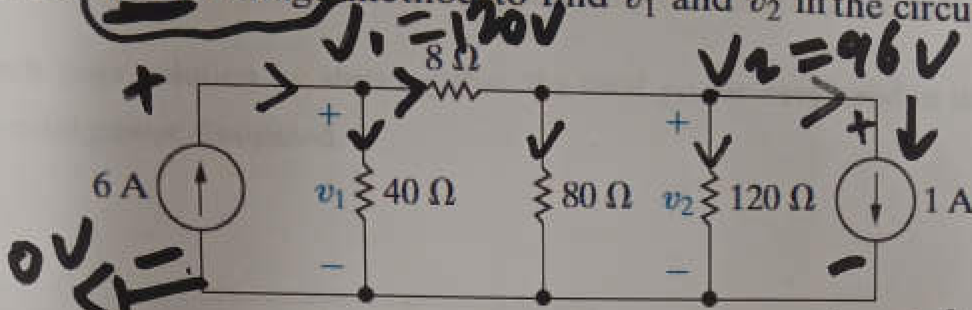
$$p_{150} = 150i^2 = 150(0.033)^2 = 0.1667\,\text{W};$$

$$p_{300} = \frac{v_o^2}{300} = \frac{15^2}{300} = 0.75\,\text{W}.$$

Thus the total power absorbed is $p_{\text{abs}} = 0.5 + 0.1667 + 0.75 = 1.4167\,\text{W}.$

Q3.

Use the node-voltage method to find v_1 and v_2 in the circuit below.



$$6 = \frac{V_1}{40} + \frac{V_1 - V_2}{8}$$

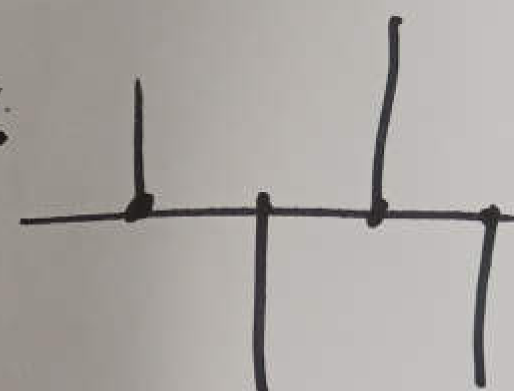
$$\frac{V_1 - V_2}{8} = \frac{V_2}{80} + \frac{V_2}{120} + 1$$

$$-6 + \frac{v_1}{40} + \frac{v_1 - v_2}{8} = 0;$$

$$\frac{v_2 - v_1}{8} + \frac{v_2}{80} + \frac{v_2}{120} + 1 = 0;$$

Solving, $v_1 = 120$ V; $v_2 = 96$ V.

CHECK:



$$p_{40\Omega} = \frac{(120)^2}{40} = 360\text{ W};$$

$$p_{8\Omega} = \frac{(120-96)^2}{8} = 72\text{ W};$$

$$p_{80\Omega} = \frac{(96)^2}{80} = 115.2\text{ W};$$

$$p_{120\Omega} = \frac{(96)^2}{120} = 76.8\text{ W};$$

$$p_{6A} = -(6)(120) = -720\text{ W};$$

$$p_{1A} = (1)(96) = 96\text{ W};$$

$$\sum p_{abs} = 360 + 72 + 115.2 + 76.8 + 96 = 720\text{ W};$$

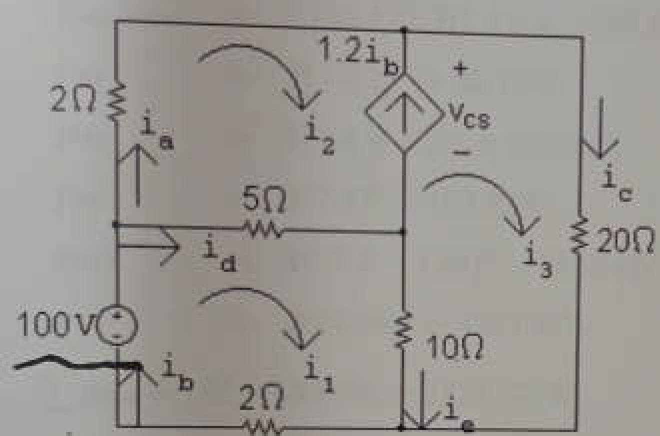
$$\sum p_{dev} = 720\text{ W} \quad \text{(CHECKS)}.$$

Q4.

a) Use the mesh-current method to find the branch currents in $i_a - i_e$ in the circuit below.

b) Check your solution by showing that the total power developed in the circuit equals the total power dissipated.

[a]



The i_1 mesh current equation:
$-100 + 5(i_1 - i_2) + 10(i_1 - i_3) + 2i_1 = 0.$
The $i_2 - i_3$ supermesh equationa:
$2i_2 + 20i_3 + 10(i_3 - i_1) + 5(i_2 - i_1) = 0.$
The supermesh constraint:
$i_3 - i_2 = 1.2i_b = 1.2i_1.$
Place these equations in standard form:

$$\begin{aligned} i_1(5+10+2) + i_2(-5) + i_3(-10) &= 100; \\ i_1(-10-5) + i_2(2+5) + i_3(20+10) &= 0; \\ i_1(1.2) + i_2(1) + i_3(-1) &= 0. \end{aligned}$$

Solving, $i_1 = 7.4\text{A}$; $i_2 = -4.2\text{A}$; $i_3 = 4.68\text{A}$.
Solve for the requested currents:

$$\begin{aligned} i_a &= i_2 = -4.2\text{A}; \\ i_b &= i_1 = 7.4\text{A}; \\ i_c &= i_3 = 4.68\text{A}; \\ i_d &= i_1 - i_2 = 11.6\text{A}; \\ i_e &= i_1 - i_3 = 2.72\text{A}. \end{aligned}$$

[b] Find v_{cs}:

$2i_2 + v_{cs} + 5(i_2 - i_1) = 0$

$\therefore \quad v_{cs} = -2(-4.2) - 5(-4.2 - 7.4) = 66.4\text{V}.$

Calculate the power:

$$
\begin{aligned}
p_{100\text{V}} &= -(100)(7.4) = -740\text{W}; \\
p_{\text{dep source}} &= -(66.4)[1.2(7.4)] = -589.632\text{W}; \\
p_{2\Omega} &= 2(-4.2)^2 = 35.28\text{W}; \\
p_{5\Omega} &= 5(7.4 + 4.2)^2 = 672.8\text{W}; \\
p_{2\Omega} &= 2(7.4)^2 = 109.52\text{W}; \\
p_{10\Omega} &= 10(7.4 - 4.68)^2 = 73.984\text{W}; \\
p_{20\Omega} &= 20(4.68)^2 = 438.048\text{W}.
\end{aligned}
$$

$\sum p_{\text{dev}} = 740 + 589.632 = 1329.632\text{W};$

$\sum p_{\text{dis}} = 35.28 + 672.8 + 109.52 + 73.984 + 438.048 = 1329.632\text{W}.$