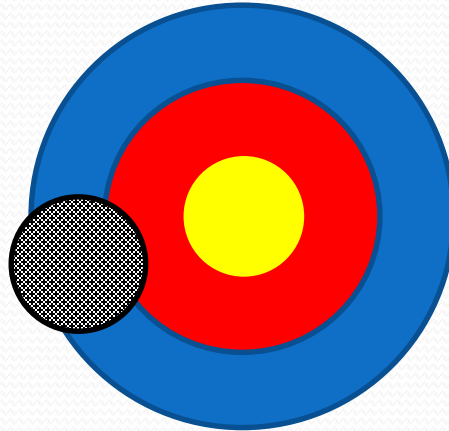
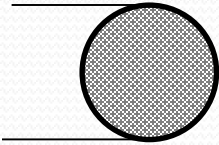


# Lecture 5

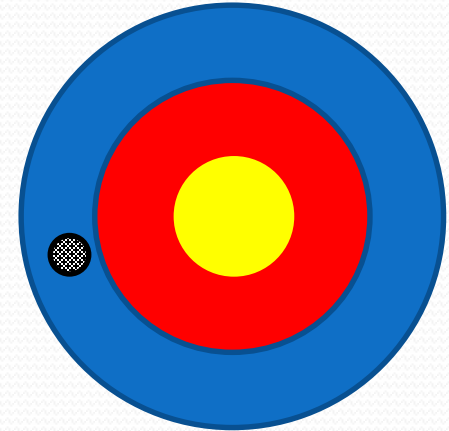
Accuracy, Precision, Significant Figures  
(Some general rules)

# Accuracy vs Precision

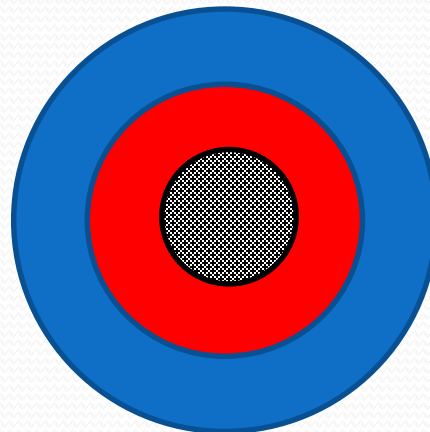
Calculation  
precision



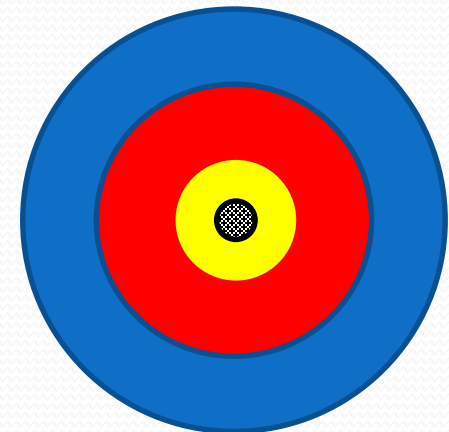
**Poor Precision**  
**Poor Accuracy**



**Better Precision**  
**Poor Accuracy**




**Poor Precision**  
**Better Accuracy**



**Better Precision**  
**Better Accuracy**

**Precision** – a measure of the uncertainty in your measurements and calculations

**Accuracy** – how close your answer is to the truth



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Accuracy: compare to the truth.  
Precision: compare to itself.

# Significant Digits

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When a number is used in any engineering context, we must be concerned with the number of significant digits that it contains. It will **affect the accuracy and precision** of any computations which use it.

## When are Digits Significant?

- Non-zero digits are always significant.
  - 22 has two significant digits
  - 22.3 has three significant digits

# Zeros are more complicated

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**A.** Zeroes placed before other digits are not significant\*;

0.046 has two significant digits

**B.** Zeroes placed between other digits are always significant;

4009 kg has four significant digits.

**C.** Zeroes placed after other digits AND behind a decimal point are significant;

7.90 has three significant digits.

*\*Unless it is 0 itself*

# Zeros

- Zeros at the end of a number are significant only if they are behind a decimal point as in case C. For example, in the number 8200, it is not clear if the zeros are significant or not. The number of significant digits in 8200 is at least two, but could be three or four. To avoid uncertainty, use scientific notation to place significant zeros behind a decimal point:

$8.200 \times 10^3$       has four significant digits

$8.20 \times 10^3$       has three significant digits

$8.2 \times 10^3$       has two significant digits

The ambiguity is removed

This isn't a mathematical law, it is more of a convention that is widely followed.

# Zero by itself is a special case

1. Treat 0. as a special case:

0. has 1 significant digit

0.0 2, zeros after the decimal are significant

0.00 3

00.00 3, zeros preceding 0. are not significant

2. When in doubt put the number into scientific notation\*, then all digits of the mantissa are significant:

0.  $0. \times 10^0$  1 significant digit

0.0  $0.0 \times 10^0$  2 significant digits

00.0  $0.0 \times 10^0$  2 significant digits

*\*ignore the fact that the strict definition of scientific notation says 0 cannot be expressed in scientific notation.*

# More examples

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1214.55	6 significant digits	
12	2	
120	2 or 3	<i>Unclear whether the 0 is meaningful or not</i>
120.	3	<i>The decimal emphasizes that the 0 is meaningful</i>
120.0	4	<i>Zeros after the decimal are significant, else you wouldn't add them.</i>
012	2	<i>Leading zeros not significant</i>
0.0012	2	<i>These are all leading zeros</i>
1.0004	5	
1.00040	6	<i>Zeros after the decimal are significant</i>



# Summary of Rules

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- Non-zero digits are always significant
- Zeros between non-zero digits are always significant
- Trailing zeros are significant if the number contains a decimal point
  - Trailing zeros are assumed to NOT be significant if the number does not contain a decimal point.
- Zeros before *significant digits* are not significant.

# Absolute Precision

- The **implied precision** is one unit of the least-significant digit's place. Without additional knowledge it has equal probability of being high or low. So we specify  $\pm\frac{1}{2}$  of a unit of the least significant digit.
- Consider a value of 10.3. We don't know what comes after the 3. If the number was rounded to 10.3, it could have been 10.25 or 10.35. The uncertainty is  $\pm 0.05$ .

Examples	Least Sig. unit	<b>Precision</b> (implied)
128.1	0.1	$\pm 0.05$
0.50	0.01	$\pm 0.005$
5.4	0.1	$\pm 0.05$

# Rules for Working with Significant Digits

- **Addition and Subtraction:** the number of digits to the right of the decimal point in the answer must be equal to the least number of digits to the right of **any of the inputs**.
- Compute the answer then round appropriately

$$\begin{array}{r} 6.778 \\ + 3.5 \\ \hline \end{array}$$

$$10.278$$

$$10.3$$

$$10.0$$

$$\begin{array}{r} 10.0 \\ - 0.0012 \\ \hline \end{array}$$

$$9.9988$$

$$10.0$$

*intermediate*

*final*

# Rules for Working with Significant Digits

- **Multiplication and Division:** the number of significant digits in the answer must equal the least number of significant digits of the input values.

$$\begin{array}{r} 7.553 \\ \times 5.52 \\ \hline \end{array}$$

41.69256

41.7

1.0

$$\div \begin{array}{r} 4.5567 \\ \hline \end{array}$$

0.21945

0.22

*intermediate*

*final*

If the result has more digits, then convert it to its scientific notation.