Multiplying Matrices

ⓐ

$$\begin{array}{cc} & \begin{matrix} c_1 & c_2 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \end{matrix} & \begin{bmatrix} 2 & -2 \\ 5 & 3 \end{bmatrix} \end{array} \times \begin{array}{cc} \begin{matrix} c_1 & c_2 \end{matrix} & \\ \begin{bmatrix} -1 & 4 \\ 7 & -6 \end{bmatrix} & \begin{matrix} r_1 \\ r_2 \end{matrix} \end{array}$$

$$\begin{array}{c} r_1 \\ r_2 \end{array} = \begin{bmatrix} \underline{r_1c_1 \cdot r_1c_1 + r_1c_2 \cdot r_2c_1} & \underline{r_1c_2} \\ \underline{r_2c_1} & \underline{r_2c_2} \end{bmatrix}$$

2×2

$$\begin{array}{cc} c_1 & c_2 \end{array}$$

$$= \begin{bmatrix} 2\times(-1)+(-2)\cdot 7 & 2\times 4+(-2)\cdot(-6) \\ 5\times(-1)+3\times 7 & 5\times 4+3\times(-6) \end{bmatrix}$$

①

$$= \begin{bmatrix} -16 & -4 \\ 16 & 2 \end{bmatrix}$$

$$8 \xrightarrow{\text{reciprocal}} \frac{1}{8}$$

$$A \xrightarrow{\text{inverse}} A^{-1}$$

Definition of inverse ~~matrix~~ matrix:

$$A \times A^{-1} = I \rightarrow \text{identity matrix}$$

$=$

inverse

Identity Matrix:

Has 1's on the diagnal and 0's everywhere else $\longrightarrow$ identity matrix

$$\underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{2\times 2} \quad \underbrace{\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}}_{3\times 3}$$

$$A \cdot I = A$$

$$\underbrace{\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}}_{A} \times \underbrace{\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}}_{I} = \underbrace{\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}}_{A}$$

$$A \times A^{-1} = A^{-1} \times A = I$$

- Sometimes, some of the matrices doesn't have an inverse matrix.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \boxed{\frac{1}{ad-bc}} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

determinant

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}^{-1} = \frac{1}{24-14} \begin{bmatrix} 6 & -7 \\ -2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix} \times \begin{bmatrix} 0.6 & -0.7 \\ -0.2 & 0.4 \end{bmatrix}$$

$$= \begin{bmatrix} 2.4-1.4=1 & -2.8+2.8=0 \\ 1.2-1.2=0 & -1.4+2.4=1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 4 \\ 6 & 7 & 8 \\ 9 & 10 & 11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$A \times X = b$$

$$A^{-1} \times A \times x = A^{-1} \times b$$

$$I \times x = A^{-1} \times b$$

$$x = A^{-1} \times b$$

Example:

$$\begin{cases} 3a+4b+5c=2 \\ 2a+6b-3c=3 \\ 7a+9b-4c=5 \end{cases}$$

$$A=\begin{bmatrix} 3 & 4 & 5 \\ 2 & 6 & -3 \\ 7 & 9 & -4 \end{bmatrix} \qquad b=\begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$A \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = inv(A) \times \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$